USE OF THE HALL EFFECT TO MEASURE VIBRATIONAL DISPLACEMENTS

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Reference [1] gives a description of certain methods of measuring small-amplitude vibrations based on the Hall effect. The present paper describes a device that can be used to measure large-amplitude vibrations over a wide range of frequencies. The requirements that must be satisfied by the magnetic system with a magnetic field varying uniformly in the direction of the vibrations are determined. Practical examples are presented.

1. Method of measurement and design equations. A Hall probe responds to the magnetic field intensity at the point in space at which it happens to be located.

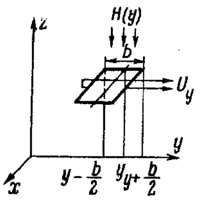

Fig. 1. Location of Hall probe in the coordinate system xyz.

Obviously, employing such a probe to measure vibrations presupposes the use of a magnetic field spatially nonuniform in the direction of displacement of the probe. If the probe moves in the direction y, the field must have a certain function H(y).

The processes in a Hall probe are given by the expression [2]

$$E_y = kE_x H. \tag{1.1}$$

Here E_x is the intensity of the longitudinal electric field in the probe, E_y is the intensity of the transverse electric field in the probe, H is the intensity of the magnetic field, and k is a proportionality factor.

Let the probe (Fig. 1) be located in the plane xy and be permeated by a magnetic field oriented along the z axis. The field is nonuniform in the y direction and is described by the expression

$$H = H(y). \tag{1.2}$$

The total output voltage of a stationary probe located in such a field is given by

$$U_{y\Sigma} = kE_x \int\limits_{y-{}^1\!/_2 b}^{y+{}^1\!/_2 b} H(y)\, dy. \tag{1.3}$$

When the probe moves in the xy plane in accordance with the law $y = f(t)$, expression (1.3) takes the form

$$U_{y\Sigma} = kE_x \int\limits_{f(t)-{}^1\!/_2 b}^{f(t)+{}^1\!/_2 b} H(y)\, dy. \tag{1.4}$$

It is obvious that

$$U_{y\Sigma} = U_{y0} + U_y. \tag{1.5}$$

Here U_{y0} is the component of the output voltage corresponding to the initial rest position of the probe; U_y is the component of the output voltage appearing in connection with the displacement of the probe.

For measuring purposes a necessary condition will be a proportional relationship between the useful signal U_y and the displacement, i.e.,

$$U_y = nf(t), \tag{1.6}$$

where n is a proportionality factor. Substituting (1.5) and (1.6) into (1.4), we obtain

$$U_{y0} + nf(t) = kE_x \int\limits_{f(t)-{}^1\!/_2 b}^{f(t)+{}^1\!/_2 b} H(y)\, dy. \tag{1.7}$$

We differentiate the left and the right sides with respect to t:

$$nf'(t) = kE_x f'(t)\{H[f(t) + {}^1\!/_2 b] - H[f(t) - {}^1\!/_2 b]\}. \tag{1.8}$$

Considering that $y = f(t)$, we obtain

$$H(y + {}^1\!/_2 b) - H(y - {}^1\!/_2 b) = \Delta H = n/kE_x. \tag{1.9}$$

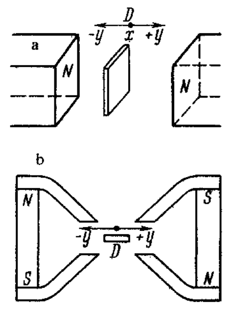

Fig. 2. Methods of creating a linearly varying magnetic field.

Equation (1.9) has the following significance. For the output voltage of the Hall probe U_y to be proportional to the displacement, the magnetic field must be such that at any y the field intensities at points located symmetrically with respect to y at a distance b/2 differ by a constant quantity ΔH. Thus, the field must vary linearly in the direction of the measured displacement

$$H = H_0 + y\,|\operatorname{grad} H| \qquad (|\operatorname{grad} H| = n/kE_x b). \tag{1.10}$$

Substituting (1.10) and (1.4), we find the total output voltage of the probe

$$U_{y\Sigma} = kE_x \int_{f(t)-1/2b}^{f(t)+1/2b} (H_0 + y\,|\,\mathrm{grad}\,H\,|)\,dy\,. \qquad (1.11)$$

Integrating and transforming (1.11), we obtain

$$U_{y\Sigma} = kE_x bH_0 + kE_x b\,|\,\mathrm{grad}\,H\,|\,f(t). \qquad (1.12)$$

As in (1.5), the first term is the rest output voltage U_y^0, and the second the useful signal proportional to the measured displacement

$$U_y = kE_x b\,|\,\mathrm{grad}\;H\,|\,f(t). \qquad (1.13)$$

Expression (1.13) can be reduced to the more convenient form

$$U_y = k_u U_x\,|\,\mathrm{grad}\;H\,|\,f(t) = k_u I_x R_x\,|\mathrm{grad}\;H\,|\,f(t). \qquad (1.14)$$

Here k_u is the so-called voltage transmission coefficient characterizing a specific Hall probe (its material and dimensions), U_x is the input voltage applied to the current electrodes of the Hall probe and creating the longitudinal field intensity E_x, R_x is the input resistance of the Hall probe, and I_x the control current.

Correspondingly, the total output voltage will be

$$U_{y\Sigma} = k_u I_x R_x [H_0 + |\,\mathrm{grad}\,H\,|\,f(t)]. \qquad (1.15)$$

From (1.14) it is easy to derive a convenient expression for determining the value of |grad H| needed to satisfy the given conditions.

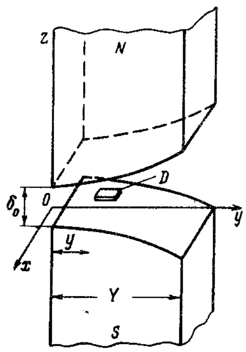

Fig. 3. Hall probe in the field of a magnet with specially shaped pole pieces.

Let, for example, $f(t)$ be sinusoidal function

$$f(t) = y_m \sin\omega t\,. \qquad (1.16)$$

The output signal must also be a sinusoidal function of the voltage with amplitude U_{ym}. Therefore, in accordance with (1.14),

$$U_{ym} = k_u U_x\,|\,\mathrm{grad}\,H\,|\,y_m, \quad \text{or} \quad |\,\mathrm{grad}\,H\,| = \frac{U_{ym}}{y_m k_u U_x} = \frac{U_{ym}}{y_m k_u I_x R_x}\,. \qquad (1.17)$$

Thus, with the help of expression (1.17) it is possible to determine the necessary magnetic field gradient for the required U_{ym} and y_m, using any available Hall probe with specific parameters k_u, I_x, and R_x.

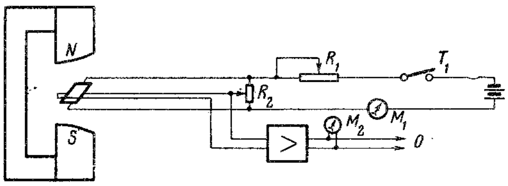

Fig. 4. Measuring circuit.

For example, suppose we have a Hall probe with the following parameters: transmission coefficient (at H = 1 Oe) $k_u = 10^{-5}$ Oe^{-1}, permissible dissipated power $P_m = 0.1$ W, and input resistance $R_x = 300$ ohms.

We shall determine the |grad H| of the field required for the probe to give an output signal with amplitude $U_{ym} = 50$ mV at a displacement amplitude $y_m = 5$ mm.

The maximum input voltage permissible with respect to dissipated power

$$U_{xm} = \sqrt{P_m R_x} = 5.5\text{ V}\,.$$

We take $U_x = 5$ V. From (1.17) we obtain |grad H| = $= 50 \cdot 10^{-3}/5 \cdot 10^{-5} \cdot 5 = 200$ Oe/mm.

A magnetic field varying uniformly in the direction of the measured displacement can be created in various ways, for example, by means of ceramic permanent magnets with facing like poles [1]. In this case a zone of constant gradient will be created in a certain region of space between the poles (Fig. 2a). A zone of constant gradient can also be created by means of a specially designed system of magnets (Fig. 2b). However, it is simplest and most convenient to create a magnetic field with constant gradient by using pole pieces of special shape (Fig. 3),* the gap between which is a variable quantity. It can be shown that to obtain a uniformly varying magnetic field with given |grad H| the gap must vary according to the law

$$\delta_y = \frac{\delta_0 Y}{Y + y\alpha_n}\,, \qquad \alpha_n = \frac{Y\,|\,\mathrm{grad}\,H\,|}{H_0}\,. \qquad (1.18)$$

Here δ_0 is the gap corresponding to the coordinate origin, δ_y the gap corresponding to a point at a distance y from the coordinate origin, Y is the width of the pole piece in the direction of the measured displacement, H_0 is the magnetic field intensity at a point corresponding to the coordinate origin, and α_n is a coefficient characterizing the total drop in field intensity along the length Y.

*I. V. Bolotin, E. A. Konstantinov, Author's Certificate No. 183416 of 4 March 1965.

This method has the additional advantage that, by using pole pieces of different length Y, it is possible to measure displacements of almost any amplitude. It is only necessary to take into account the influence of the edge effect. In practice, the relationship $Y \geq \geq 2y_m + 3$ (in mm) should hold between the amplitude of the probe displacement y_m and the width of the pole piece.

2. Measuring circuit. The measuring circuit is shown in Fig. 4. In the magnet gap the Hall probe D is attached to the object whose displacement is to be measured. The plane of the Hall probe is perpendicular to the vector of the field intensity H. The Hall probe is connected to the measuring device by a four-wire cable. The probe control current, which regulates its sensitivity, is adjusted by means of the rheostat R_1 and the milliammeter M_1. Potentiometer R_2 is used for adjusting the equipotentiality of the Hall probe. This potentiometer is also used to compensate the output voltage component U_{y0} corresponding, in accordance with (1.5) and (1.12) to the initial rest position of the probe. The output voltage obtained with the Hall electrodes of the probe, which is proportional to the instantaneous value of the vibrational displacement, is fed to a loop oscillograph either directly or, in the case of a small signal, across an amplifier. The vibration amplitude can be measured by means of an indicating instrument M_2.

3. Compensation of parasitic emf induced in the Hall probe circuits upon displacement in a magnetic field. In a Hall probe moving in a magnetic field there is generated not only a transverse Hall emf, the useful signal proportional to the displacement, but also a parasitic induction emf. The magnitude of the latter is of the same order as the useful Hall emf and depends on the geometrical dimensions of the probe, the design of the circuits connected to its electrodes, and the rate of displacement of the probe in the magnetic field. There are two Hall probe circuits: the output circuit and the control current circuit; the voltage at the output electrodes of a probe moving in a nonuniform magnetic field with constant gradient is represented, with allowance for parasitic induced emf, by

$$U_{y\Sigma} = k_u R_x I_x H_0 + k_u R_x I_x y \,|\,\text{grad}\, H\,| - k_u \frac{R_x}{R_1} s_1 H_0 \,|\,\text{grad}\, B\,| \frac{dy}{dt} - k_u \frac{R_x}{R_1} s_1 \,|\,\text{grad}\, H\,|\,|\,\text{grad}\, B\,|\, y \frac{dy}{dt} - s_2 \,|\,\text{grad}\, B\,| \frac{dy}{dt}. \quad (3.1)$$

Here $|\text{grad}\, B| = \mu_0 |\text{grad}\, H|$ is the induction gradient in the gap, μ_0 is the permeability of air, s_1 is the area of the control current circuit loop, s_2 the area of the output circuit loop, and R_1 the total resistance of the control current circuit.

Expression (3.1) consists of five terms: the first represents the constant rest component which is compensated in adjusting the equipotentiality, the second is the useful signal, while the other terms represent parasitic induced emf's ΔU proportional to the rate of displacement.

When $y = y_m \sin \omega t$, after transformations, the expression for the output voltage (disregarding the first constant component) takes the form

$$U_{y\Sigma} = U_{ym} (\sin \omega t - A_1 \cos \omega t - A_2 \cos \omega t - A_3 \sin 2\omega t) \quad (3.2)$$

$$A_1 = \frac{s_2 \omega \mu_0}{k_u R_x I_x}, \quad A_2 = \frac{s_1 \omega B_0}{I_x R_1}, \quad A_3 = \frac{s_1 \omega \,|\,\text{grad}\, B\,|\, y_m}{2 I_x R_1}, \quad U_{ym} = k_u I_x R_x \,|\,\text{grad}\, H\,|\, y_m, \quad (3.3)$$

i.e., the induced emf consists of ordinary and double-frequency components relative to the vibration frequency. We shall evaluate the quantities A_1, A_2, A_3 for a probe and a magnetic field with the parameters given in the example. We take $R_1 \approx R_x$, $I_x R_x = U_x = 5$ V, $k_u = 10^{-5}$ Oe^{-1}, $|\text{grad}\, B| = 200$ G/mm $= 200 \cdot 10^{-8}$ V $\cdot$ sec/cm^2 $\cdot$ mm, $\mu_0 = 0.4\pi \cdot 10^{-8}$ H/cm, $B_0 = 6000$ G $= 6000 \cdot 10^{-8}$ V $\cdot$ sec/cm^2, $y_m = 5$ mm.

For these parameters we obtain (s_1 and s_2 in cm^2, $\omega = 2\pi f$ in sec^{-1})

$$A_1 = 2 \cdot 10^{-4} s_2 \omega, \quad A_2 = 12 \cdot 10^{-6} s_1 \omega, \quad A_3 = 1 \cdot 10^{-6} s_1 \omega. \quad (3.4)$$

Since s_1 and s_2 are the quantities of the same order, analysis of expression (3.4) shows that the components A_2 and A_3 can be neglected as compared with A_1 and the useful signal, i.e., it is necessary to take into account only the induced emf for the output circuit.

An analysis of expression (3.3) shows that in order to reduce the influence of the induced emf it is desirable to use high-resistance Hall probes with a large voltage transmission coefficient. The electrical circuits from the electrodes should be bifilar, with minimum areas s_1 and especially, s_2. In this case at frequencies of the order of 50–100 Hz the induced emf can be neglected. At higher vibration frequencies the induced emf may be intolerably large, since they are proportional to the frequency. In this case it is necessary to take measures to compensate the parasitic emf.

The output circuit parasitic emf can be successfully compensated by a counterturn, whose area is selected experimentally in constructing the instrument. The current circuit parasitic emf are suppressed by using a large resistance R_1 with a corresponding increase in supply voltage. An example of such compensation is given in [3].

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